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Authors’ addresses:

Oliver Ruhnau
RWTH Aachen University
Templergraben 55
52056 Aachen, Germany
E-Mail: oliver.ruhnau@rwth-aachen.de

Patrick Hennig
Grundgrün Energie GmbH
Uhlandstraße 181 / 183
10623 Berlin, Germany
E-Mail: post@patrick-hennig.de

Reinhard Madlener
Institute for Future Energy Consumer Needs and Behavior (FCN)
School of Business and Economics / E.ON Energy Research Center
RWTH Aachen University
Mathieustrasse 10
52074 Aachen, Germany
E-Mail: RMadlener@eonerc.rwth-aachen.de

Publisher: Prof. Dr. Reinhard Madlener
Chair of Energy Economics and Management
Director, Institute for Future Energy Consumer Needs and Behavior (FCN)
E.ON Energy Research Center (E.ON ERC)
RWTH Aachen University
Mathieustrasse 10, 52074 Aachen, Germany
Phone: +49 (0) 241-80 49820
Fax: +49 (0) 241-80 49829
Web: www.eonerc.rwth-aachen.de/fcn
E-mail: post_fcn@eonerc.rwth-aachen.de
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Oliver Ruhnau\textsuperscript{1,*}, Patrick Hennig\textsuperscript{2}, and Reinhard Madlener\textsuperscript{3}

\textsuperscript{1}RWTH Aachen University, Templergraben 55, 52056 Aachen, Germany
\textsuperscript{2}Ahlbeckerstr. 18, 10437 Berlin, Germany
\textsuperscript{3}Institute for Future Energy Consumer Needs and Behavior (FCN), School of Business and Economics / E.ON Energy Research Center, RWTH Aachen University, Mathieustr. 10, 52074 Aachen, Germany

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Abstract
Forecasts are usually evaluated in terms of accuracy. With regard to application, the question arises if the most accurate forecast is also optimal in terms of forecast related costs and risks. Combining insights from research and practice, we show that this is indeed not necessarily the case. Our analysis is grounded in the dynamic field of short-term forecasting of solar electricity feed-in. A clear sky model is implemented and combined with a linear model, an autoregressive model, and an artificial neural network. These models are applied to a portfolio of ten large-scale photovoltaic systems in Germany. We compare the different models in order to quantify the connection between errors and costs. We find that apart from accuracy, correlation with market prices is an important characteristic of forecasts when economic implications are considered as important.

Keywords: Forecasting evaluation, renewable energy, electricity markets, balancing costs, artificial neural networks, clear sky model, Germany.

*Corresponding author. E-mail: oliver.ruhnau@rwth-aachen.de
1 Introduction

The combination of governmental incentives and falling investment costs has led to a rapidly growing share of electricity generated from solar and wind energy. In Germany, for example, renewable energies temporarily met 80% of the electricity demand in 2014 (Graichen et al. 2015). Consequently, these intermittent renewable energy sources become more and more important to the electricity system, including the challenge of handling their volatility. In this context, accurate and reliable short-term forecasts of power generation are required for both electricity trading and grid operation.

Several recent studies present forecasting models for solar and wind feed-in using fundamental equations, statistical methods, artificial intelligence techniques, or a combination of these approaches. Thereby, research has a strong focus on accuracy which becomes manifest most clearly in electricity forecasting competitions (e.g. Hong et al. 2014) and benchmarking studies (e.g. Lorenz et al. 2009). Unfortunately, the relative performance ranking of methods varies with the accuracy measure (Crone et al. 2011). To overcome this shortcoming with regard to the right choice of forecasts, some studies are specifically dedicated to discuss the right choice of an error measure (e.g. Madsen et al. 2005; Kostylev and Pavlovski 2011).

However, comparatively few studies analyze the economic implications of such forecasts. Across different renewable energy sources and market conditions, they prove a clear positive value of forecasting (e.g. Parkes et al. 2006; Kraas et al. 2013). This value is usually estimated by comparing market revenues of using an advanced forecast to reference scenarios, such as naive habit persistence or not trading at all.

It can be expected that enhanced accuracy of a forecast has positive economic implications. This idea is supported by a sensitivity analysis of simulated forecasts of different predefined error levels, as it was carried out for Spain by Barthelmie et al. (2008). However, this result cannot be generalized, because economic implications are dependent on the market conditions and, more specifically for the case of electricity markets, on how the costs of forecast errors are determined. In some electricity markets, such as the Spanish one, every unit of deviation causes a negative economic impact which distinctly links it to accuracy.

In other electricity markets, such as the German one, the economic implications of forecast errors, which are referred to as balancing costs, are indeed calculated as the product of the forecast error and the spread between the prices in the relevant electricity markets (von Roon 2011; Garnier and Madlener 2015). Depending on the sign of both

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1 On May 11 which was the day with the maximum electricity feed-in from renewable energy sources in 2014.
variables, economic implications can be both positive or negative. Even though the expected value of errors and prices should be around zero\(^2\), economic implications of forecast errors are more likely to be negative and cause significant costs to companies engaged in renewable electricity marketing. This is due to the influence that forecast errors have on market price differences and the resulting correlation between both variables (Hirth and Ziegenhagen 2015).

This study elaborates on the non-trivial connection between forecast accuracy and economic implications for the case of large-scale photovoltaic (PV) systems in Germany\(^3\). To this end, we implement and compare different well-known models for day-ahead forecasting of PV electricity feed-in. More specifically, a linear model (LM), an autoregressive model with exogenous input (ARX), and an artificial neural network (ANN) are analyzed in turn. All models include inputs from numerical weather prediction (NWP) which have been retrieved from ECMWF\(^4\), because a benchmarking study has shown the best results for this NWP provider (Lorenz et al. 2009). Furthermore, every model is combined with a statistical clear sky model (Bacher et al. 2009) which estimates the electricity feed-in under clear sky conditions. The resulting hybrid models generate forecasts of different accuracy which provides useful for our analysis. However, note that more complex models, including e.g. those suggested by Fernandez-Jimenez et al. (2012), probably achieve a higher accuracy.

We evaluate the accuracy of the different forecasts by calculating the mean average error (MAE). Following decision theory, economic implications are quantified using a two-dimensional mean-variance approach. This accounts for the fact that financial risk plays an important role in renewable electricity marketing practice. From both theory and empirical results, we derive formulae linking forecast accuracy with cost implications. We show that the latter strongly depends on the correlation between electricity forecast errors and market prices and, eventually (at least to some extent), on how errors of various forecasts correlate among themselves.

The remainder of this paper is structured as follows. Section 2 describes the set-up of our study as well as the forecasting models and the evaluation methods applied. Section 3 presents the results of our investigation, which are then summarized and discussed in section 4. Section 4 also concludes.

\(^2\)In case the expected value of the forecast errors is significantly different from zero, i.e. if the forecast is biased, one could simply improve it by subtracting the bias from future forecasted values. If the expected value of the price spread between two electricity markets is not zero, some arbitrage opportunity exists.

\(^3\)Note that other (European) electricity markets feature similar conditions.

\(^4\)European Centre for Medium-Range Weather Forecasts, an independent intergovernmental organization supported by 34 stated and based in Reading, UK.
2 Methodology

2.1 Subject, Data and Context of Investigation

Our analysis is based on a portfolio of ten large-scale PV systems\(^5\) installed all over Germany with a total capacity of 156.7 MW. Feed-in time series from 2014 are available for those systems in a quarter-hourly resolution. Time series for 2013 are additionally available for PV systems 6 and 10. For each quarter-hour of the year, there is a value indicating how much electricity [kWh] has been fed into the grid.

Our forecasting models use NWP for the parameters “solar irradiance” and “clear sky irradiance” as input. For testing, two historical NWPs from the global atmospheric reanalysis model *ERA-Interim* of ECMWF are available per day, starting from the initial points 00:00 UTC and 12:00 UTC. For the purpose of day-ahead electricity forecasting, NWPs from 00:00 UTC of the previous day are of interest as 12:00 UTC is after gate-closure of the day-ahead electricity market which is at 11:00 (10:00) UTC for winter (summer) time. Operating at the global scale, the ERA-Interim model generates data at low spatial (about 28 x 17 km) and temporal resolution (6 hours). Hence, we apply interpolation to generate site-specific time series at a quarter-hourly resolution. More detailed information about the ERA-Interim reanalysis model of ECMWF can be found in Berrisford et al. (2009).

The economic impacts of different feed-in forecasts are analyzed in the context of German electricity markets, more precisely the day-ahead and the intra-day market. Solar electricity is sold in the day-ahead auction, where bids for each delivery hour of the next day are proposed based on day-ahead feed-in forecasts. After the gate closure at 12:00 (local time), those bids are ranked for each hour in ascending order of price. Hourly market clearing prices and corresponding delivery schedules are derived from the intersection of the resulting merit order supply curve with demand. Intra-day trading is possible from 3 pm (day-ahead) until 45 minutes before delivery. It allows market participants to take corrective actions on these schedules, e.g. if intra-day solar feed-in forecasts deviate from day-ahead forecasts. Note that errors occurring after intra-day gate closure are balanced in the imbalance market. However, as we consider day-ahead forecasts only, this is beyond the scope of our study.

The correlation of forecasting errors is examined using a commercially available reference forecast of which we know that it is used by several companies engaged in solar electricity marketing (direct marketers as well as transmission system operators) in Germany.

\(^5\)Ranging from 0.8 MW to 82 MW of installed peak capacity.
2.2 Forecasting Models

The electricity output of a solar PV system is a function of (1) deterministic factors, such as the sun’s position and the collector’s orientation, and (2) stochastic weather impacts, such as cloud cover, aerosols, fog, snow, and temperature. In this study, deterministic and stochastic factors are modeled separately by means of a clear sky approach. To this end, the actual (forecasted) solar irradiance \( I^{(DA)}_t \) is defined as

\[
I^{(DA)}_t = f^{CSI}_t I^{CS}_t,
\]

where \( f^{CSI}_t \) is the (stochastic and unit-less) clear sky irradiance factor and \( I^{CS}_t \) is the (deterministic) irradiance under clear sky conditions. Similarly, the actual (forecasted) electricity feed-in is written as

\[
E^{(DA)}_t = f^{CSF}_t E^{CS}_t,
\]

where \( f^{CSF}_t \) is the clear sky electricity factor and \( E^{CS}_t \) is the clear sky electricity feed-in. Using these definitions, both clear sky factors are high for clear and low for overcast days\(^6\). Note that even under clear sky conditions, the electricity feed-in is weather-dependent due to temperature effects on the PV system’s efficiency. However, this effect is rather small\(^7\) and thus can be safely neglected when calculating \( E^{CS}_t \).

The next subsection describes how the electricity under clear sky conditions is calculated. Thereafter, three different forecasting models for the clear sky electricity factor are introduced (the LM, the ARX, and the ANN).

2.2.1 Clear Sky Model

Calculation of the clear sky electricity feed-in can follow either the fundamental or a statistical approach. In this study, an advanced statistical clear sky model based on Bacher et al. (2009) is implemented. Calculations use the actual feed-in time series \( E_t \) as presented in section 2.1. This time series can be rearranged such that

\[
E_t = E(x_t, y_t),
\]

where \( x = 1, \ldots, 365 \) is the day of the year and \( y = 1, \ldots, 96 \) is the quarter-hour of the day. The clear sky electricity time series can be seen as the upper surface of the resulting point cloud and can be estimated by statistical methods. In this study, we apply weighted

\(^6\)Ideally, the clear sky factors should take values between zero and one. In our case, however, they can also be slightly above one due to the statistical approach that has been applied (Chapter 2.2.1).

\(^7\)According to Marion et al. (2005), the so-called power correction factor amounts to -45%/K.
quantile regression as described in Bacher et al. (2009), except for two refinements that we developed in order to treat two shortcomings of the original approach. Firstly, the “day of the year”-distance function is modified in order to address scarcity of clear sky winter days. Secondly, a simple correction tool deals with systematic overestimation of the clear sky electricity feed-in around the start and the end of the daily feed-in. The refinements are described in detail in Appendices A and B, respectively. The rearranged time series of actual and clear sky values are shown in Fig. 1.

![Figure 1: Actual (left) and clear sky (right) electricity feed-in from PV system 10 for the year 2014. The values are given in kWh/quarter-hour and were rearranged by the day of the year (DoY) and the quarter-hour of the day (QHoD).](image)

Bacher et al. (2009) use the same time series of actual values as input to the clear sky estimation and for the model evaluation, which seems unsatisfactorily with regard to the forecasting application considered here. Results might be biased by yearly particularities and thus enhance the forecasting performance compared to a true out-of-sample forecast. We investigate this hypothesis using the example of the PV systems 6 and 10 where feed-in time series are available for 2013 and 2014. The analysis of the resulting clear sky electricity time series reveals significant inter-yearly differences. However, very similar forecast accuracies can be achieved with both time series. Apparently, the integrated model architecture is able to balance inter-yearly differences. Given these results, in the following, we use actual values for 2014 as input to the clear sky estimation (as those values are available for all PV systems) and it is assumed that the same forecast accuracy could be achieved if using actual values for 2013 (which would be feasible in practice).

---

8Those differences can be explained by weather influences. In case there are many (few) clear sky days in the environment of an estimation point, this will increase (decrease) the estimation result of the quantile regression. For example, comparatively low clear sky values in spring 2013 can be explained by a long lasting period of snow cover in this year.

9If the clear sky model is combined with a linear model, for example, regression coefficients during winter are significantly higher when using (low winter) clear sky values from 2013.
2.2.2 Linear Model

The LM can be described by

\[ f_{t}^{CSE} = \beta_0 + \beta_1 f_{t}^{CSI} + \epsilon_t, \]  

(4)

where \( f_{t}^{CSI} \) is obtained from NWP, \( \beta_0 \) and \( \beta_1 \) are regression coefficients, and \( \epsilon_t \) is the residual term, which is assumed to be normally distributed. For each forecasting day, coefficients are estimated by ordinary least squares regression based on quarter-hours from a fixed number of preceding days (30), where the actual feed-in exceeded a given threshold (0.2% of installed-capacity-equivalent feed-in). Thus, the model features seasonality and very unproductive quarter-hours are excluded from the regression, because both \( f_{t}^{CSE} \) and \( f_{t}^{CSI} \) do not take reasonable values in that case.

2.2.3 Autoregressive Model with Exogenous Input

The ARX can be characterized by

\[ f_{t}^{CSE} = \beta_0 + \beta_1 f_{t}^{CSI} + \beta_2 f_{(t-d)}^{CSE} + \epsilon_t, \]  

(5)

where \( f_{(t-d)}^{CSE} \) is the most recent observation of the clear sky electricity factor at the same time of the day which is available for forecasting. Regression coefficients are computed based on weighted least squares with exponential forgetting (forgetting factor 0.999) as presented in Bacher et al. (2009). Again, a threshold is applied in connection with actual feed-in (same value as for the LM).

2.2.4 Artificial Neural Network

The ANN that we use in our study is of the Multilayer Perceptron (MLP) type. It consists of one input neuron (clear sky irradiance factor), one output neuron (clear sky electricity factor), and one layer of hidden neurons featuring sigmoid activation functions. Connection weights and biases of the MLP are determined with the help of a back propagation training algorithm. A good description of the structure of an MLP and of the back propagation concept in the context of PV feed-in forecasts can be found in Mellit and Kalogirou (2008).

In order to avoid over-fitting of the MLP, a cross-validation algorithm has been developed to select the optimal number of hidden neurons, the initial weights, and the number of iterations for back propagation. The algorithm randomly assigns input and output

\[ The values that have been chosen show the best results in our sensitivity analyses.\]
values from the 30 most recent days to two disjunct data sets, one for the training (70% of the data) and one for the cross-validation (30% of the data). Several MLPs are trained on the training data set with different parameters and initial weights. When the training is finished after a predefined number of iterations, the network error is calculated for the cross-validation set. The MLP with the lowest cross-validation error is chosen for the forecasting.

For every period of ten days, 2160 new MLPs are trained and the best one is selected by cross-validation\textsuperscript{11}. As results might be dependent on the composition of training and cross-validation data, the random creation of data sets is repeated several times, a procedure which we refer to as “cross-validation loop”. Table 1 summarizes cross-validation and back propagation parameters that, according to our sensitivity analyses, generated good and robust results.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Selected value(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cross-validation loops</td>
<td>12</td>
</tr>
<tr>
<td>Number of hidden neurons</td>
<td>({1, 3, 6, 10})</td>
</tr>
<tr>
<td>Number of random sets of initial weights</td>
<td>20</td>
</tr>
<tr>
<td>Number of training iterations</td>
<td>({10, 30, 60, 100})</td>
</tr>
<tr>
<td>Learning rate</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\textbf{2.3 Evaluation of Forecast Accuracy}

Following Kostylev and Pavlovski (2011), we take the forecasting purpose into account when choosing an appropriate error measure. In our case, forecasts are used for the (profit-maximizing) trading of solar electricity at the day-ahead electricity market, and every unit of deviation from the resulting schedule causes a (likely negative) economic impact. Consequentially, the MAE is calculated by

\[
MAE = \frac{\sum |E_{t}^{DA} - E_{t}|}{\sum E_{t}},
\]

where the sums include values from all quarter-hours of a given period. To allow for a comparison among PV systems of different size, the values are normalized with respect to actual electricity generation, which has been chosen because it is most closely related to profits gained from energy trading and thus best reflects the forecasting purpose.

\textsuperscript{11}12 cross-validation loops x 4 different numbers of hidden neurons x 20 random sets of initial weights = 2160 MLPs in total.
2.4 Evaluation of Economic Implications

Balancing costs \((BC_t)\) are usually calculated by comparing the actual revenues and costs of marketing and balancing activities to what would have been the marketing revenues in the (hypothetical) case of a perfect forecast. This leads to the following equation

\[
BC_t = (E_t - E_{t}^{DA})P_{t}^{DA} + (E_{t}^{DA} - E_{t}^{ID})P_{t}^{ID} + (E_{t}^{ID} - E_{t})P_{t}^{BA},
\]

(7)

where \(E_t\) is the actual electricity feed-in, \(E_{t}^{DA}\) and \(E_{t}^{ID}\) are the day-ahead and intra-day feed-in forecasts, \(P_{t}^{DA}\) and \(P_{t}^{ID}\) are electricity prices at the day-ahead and intra-day markets, and \(P_{t}^{BA}\) is the imbalance price (reBAP). Note that \(BC_t > 0\) is related to costs and \(BC_t < 0\) is related to earnings through balancing efforts (cf. von Roon 2011). This study presents a day-ahead electricity forecast only, i.e. no intra-day forecast is available for balancing cost calculations. For this reason, balancing costs are rewritten as

\[
BC_t = (E_{t}^{DA} - E_{t})(P_{t}^{ID} - P_{t}^{DA}) + (E_{t}^{ID} - E_{t})(P_{t}^{BA} - P_{t}^{ID}),
\]

(8)

where the first part of the equation is dependent on the day-ahead forecast, and the second part is dependent on the intra-day forecast. In the following, we will only consider the part with the dependency on the day-ahead forecast, which is equivalent to assuming the intra-day forecast to be perfect:

\[
BC_t = (E_{t}^{DA} - E_{t})(P_{t}^{ID} - P_{t}^{DA}) = \Delta E_t \Delta P_t.
\]

(9)

Intra-day prices are highly time-dependent, and intra-day balancing costs can be optimized through strategic balancing behavior (Garnier and Madlener 2015). To exclude such influences from our analysis, the intra-day reference price is used, which is the volume-weighted average price of all deals that are related to a given quarter-hour and that were realized during the last 15 minutes before the respective intra-day gate-closure. Thus, we assume that deviations are balanced right before gate-closure, at an average price which is in line with the reality of the German intra-day market. Furthermore, our analysis ignores the option to a priori define a portion of the day-ahead forecasted electricity that is sold at the intra-day market for strategic reasons.

According to the main principle of decision theory we assume forecast-users to aim at minimizing their expected costs and risk. In order to estimate those two dimensions for a given forecast, we calculate the mean and the standard deviation of daily balancing costs. To allow for a comparison among PV systems of different size, both values are normalized using the installed capacity and then referred to as the relative mean of daily costs (rMDC) and the relative standard deviation of daily costs (rSDC), respectively.
2.5 Connection between Accuracy and Economic Implications

From the definition of the covariance, it can be derived that

\[ E(BC_t) = E(\Delta E_t \Delta P_t) = E(\Delta E_t) E(\Delta P_t) + cov(\Delta E_t, \Delta P_t), \]  

where \( E \) and \( cov \) denote the expected value and the covariance, respectively. As argued in the introduction, \( E(\Delta E_t) \approx 0 \) and \( E(\Delta P_t) \approx 0 \). Thus, we can rewrite Eq. (10) as:

\[ E(BC_t) \approx cov(\Delta E_t, \Delta P_t) = cor(\Delta E_t, \Delta P_t) sd(\Delta E_t) sd(\Delta P_t) \equiv A_1, \]  

where \( cor \) and \( sd \) are the correlation and the standard deviation. From this fundamental equation, we can see that balancing costs are dependent on both the forecast accuracy, represented by \( sd(\Delta E_t) \), and the correlation between forecast errors and price spreads\(^{12}\).

With regard to previous considerations, we aim at quantifying the forecast accuracy in terms of the MAE. In the following, we show empirically that

\[ E(BC_t) \sim cor(\Delta E_t, \Delta P_t) MAE^2 \equiv A_2 \]  

approximates the connection between balancing costs and forecast accuracy adequately. We name the fundamental term and the empirical term as \( A_1 \) and \( A_2 \), respectively. Note that again we use the rMDC to quantify the expected balancing costs (\( rMDC \sim E(BC_t) \)).

3 Results

The upper plot in Fig. 2 shows the forecast accuracy for the considered PV systems and the portfolio. Apparently, the ARX always has the lowest MAE, followed by the LM and the ANN. As expected, due to spatial averaging of forecasting errors, the MAE of the portfolio is remarkably lower than for single systems.

Overall, the forecast-related risk behaves similarly: As displayed by the lower plot in Fig. 2, the rSDC of the portfolio is lowest for the ARX, average for the LM, and highest for the ANN. For single systems, however, there are some variances for the ANN.

The center plot in Fig. 2 compares the rMDC for the different forecasting models. Apparently, for the portfolio and for the majority of PV systems, balancing costs are lowest for the LM, even though forecast accuracy is higher for the ARX.

According to section 2.5, the correlation between forecasting errors and market prices

\(^{12}\)Balancing costs are also dependent on the volatility of market prices \( sd(\Delta P_t) \), but those do not differ with regard to different forecasting models.
Figure 2: Forecast accuracy and economic implications for the considered PV systems and the respective portfolio (PF).

should be able to explain this phenomenon. Fig. 3 shows the ratio between $A_1$ (Eq. 11) and the rMDC (left plot) as well as the ratio between $A_2$ (Eq. 12) and the rMDC (right plot) for the different forecasting models. In order to facilitate the inspection, the ratios have been normalized for each single system such that the ratio of the ANN is equal to unity.

For the majority of single systems, the ratios are approximately constant across different forecasting models. Thus, we can conclude that both the theoretical Eq. (11) and the empirical Eq. (12) hold true (the latter connection is even more distinct in the data). The two outliers are from PV systems 2 and 3 and probably due to the fact that their rMDC which are in the denominator of our ratios are so small.

Hence, the low economic performance of the ARX is caused by a high error-price correlation which buys out its enhanced accuracy. Indeed, this is confirmed by the upper plot in Fig. 4 which reports the correlation between the forecasting errors of our models and the price spread between the day-ahead and the intra-day market. This correlation is generally higher for the ARX than for the other two models.

As already stated in the introduction, the forecast errors of the market participants as a whole have an influence on market prices. Therefore, the hypothesis arises that the correlation of the enhanced forecasting model might be higher due to the fact that it is more similar to the forecasting models that are used by the majority of market
participants. In order to quantify this similarity, we calculate the correlation between the forecast errors of our models and those of the commercial reference forecast introduced in section 2.1. As shown in the lower plot of Fig. 4, this correlation is indeed higher for the ARX than for the other models considered.

This supports our idea that forecast similarity increases price correlation and eventually decreases economic performance. Note, however, that price correlation cannot be explained by similarity only. For instance, the ANN features a comparatively high price correlation even though the correlation with the reference model is quite low for the majority of PV systems, and also for the portfolio overall.

Figure 4: Correlation of our models’ forecast errors with market prices (upper plot) and with the forecast errors of a commercial forecast (lower plot) for the single systems and for the overall portfolio (PF).
4 Discussion and Conclusion

In this paper, we compared different solar electricity feed-in forecasting models in the context of German market conditions. Our investigation leads to the conclusion that enhanced forecast accuracy clearly reduces the forecast-related financial risk. It also has a decreasing impact on the expected value of forecast-related costs. However, our study proves that the latter is also strongly dependent on the correlation between forecasting errors and market price differences. Consequently, it is possible that a more accurate forecasting model entails adverse economic implications, a counterintuitive result. In light of these findings, commonly used unidimensional accuracy-based forecast assessment remains incomplete and thus suboptimal.

The insights from our analysis can be used for both forecast development and application. Our investigation suggests that correlation is dependent on the similarity of forecasts. Research should elaborate on this connection and search for models which at the same time feature high accuracy and low correlation. Concerning forecasting practice, our findings contribute to the discussion on which error measure the choice of the right forecast should be based. Our approach can be merged with decision theory (considering risk aversion) for calculating the utility of a forecast which can serve as the one-and-only performance parameter instead of an error measure. In order to ensure a reliable decision, further work should quantify the uncertainty of the error-price correlation, i.e. how much this value changes over time for one specific forecasting model.

Hence, forecasting assessment boils down to a trade-off between accuracy and correlation. However, it is important to be aware of the limits to this idea. So far, we have assumed that the prices are independent from the choice of the forecasting model. In reality, this assumption does not hold true. On the one hand, there is an incentive to avoid very large errors of the own forecast, as these would disadvantageously affect prices. On the other hand, the optimal forecast choice of one market participant is dependent on which forecasting models the other participants choose. The following example illustrates this idea: Imagine there are two forecasts of the same accuracy. Whenever a company switches from forecast A to B, the correlation between those forecasts and prices will decrease and increase, respectively. In the end, it can be expected that a Nash equilibrium will be reached where both forecasts are in use.

Finally, there is an incentive for both forecast accuracy and diversity. Our analysis focuses on the very specific (but important) field of solar electricity feed-in forecasting in the German market context. Nevertheless, the results are also applicable to wind electricity forecasts as well as to other markets that feature similar conditions.
more general level, our findings might even serve as an impulse to forecasting studies in other applications that have so far exclusively focused on accuracy.

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References


Appendix A: Clear Sky Model Weight Assignment

In order to estimate clear sky electricity generation, local weights are assigned by means of a two-dimensional smoothing kernel function in such a way that the influence of the observation at \((x_i, y_i)\) is decreasing with increasing distance from the estimation point \((x_t, y_t)\). The local weighting function for observations is defined as

\[
k(x_t, y_t, x_i, y_i) = \frac{w(x_t, x_i, h_x)w(y_t, y_i, h_y)}{\sum_i w(x_t, x_i, h_x)w(y_t, y_i, h_y)},
\]

where

\[
w(x_t, x_i, h_x) = f_{std}(\text{dist}(x_t, x_i)/h_x)
\]

and

\[
w(y_t, y_i, h_y) = f_{std}(\text{dist}(y_t, y_i)/h_y)
\]

are Gaussian kernel functions of each dimension, \(h_x\) and \(h_y\) determine the width of the weighting kernel, and \(f_{std}\) is the standard normal probability density function. The distance functions are specified as

\[
\text{dist}(x_t, x_i) = \min\{|x_t - x_i|, ||x_t - x_i| - 365|\}
\]

and

\[
\text{dist}(y_t, y_i) = |y_t - y_i|
\]

for the two dimensions “day of year” and “quarter-hour of day”, respectively.

Compared to Bacher et al. (2009), the “day of year distance function has been refined. With the new distance function, the first day of the year is defined to succeed the last day of the year. Thus higher weights are assigned to end-of-year actual values when calculating start-of-year clear sky values, and vice versa. This procedure arises from fundamental similarities between the end-of-year and start-of-year values and is supposed to generate more robust results for this season as more actual values are taken into account for the clear sky calculation.

The resulting smoothing kernel is shown in Fig. 5 for \(h_x = 35\) and \(h_y = 0.8\), which has been identified as the optimal parametrization by Bacher et al. (2009).
Figure 5: Smoothing kernel with refined day of year distance function for parameters $h_x = 35$ and $h_y = 0.8$ at the position $x_t = 20$ and $y_t = 40$.

Appendix B: Correction Procedure for the Start and the End of the Daily Feed-in

The clear sky model as presented by Bacher et al. (2009) systematically overestimates the clear sky electricity feed-in around the start and the end of the daily feed-in. This appendix presents a simple correction procedure for this bias. In a first step, the quarter-hour of the (actual) start of the daily feed-in is defined as

$$s_x = \min\{y | E(x,y) > 0\} \quad \forall x = 1 \ldots 365 \quad (18)$$

whereas the quarter-hour of the (actual) end of the daily feed-in is defined as

$$e_x = \max\{(yE(x,y) > 0\} \quad \forall x = 1 \ldots 365 \quad (19)$$

for each day of the year $x$. The results are shown in Fig. 6 (gray points). On this basis, the following estimation algorithm computes the estimated start of the daily feed-in, $\hat{s}_x$, from the recent start of the daily feed-in observations, $s_x$:

1. Initial estimation at winter solstice (around December 21), i.e. at $x = 355$:

$$\hat{s}_{355} = \min\{s_{344}, s_{345}, \ldots, s_{354}\} \quad (20)$$

2. Estimation from after winter solstice until summer solstice (around June 21), i.e. at $x = 356, \ldots, 365, 1, \ldots, 172$:

$$\hat{s}_x = \min\{\hat{s}_{x-1}, s_{x-1}\} \quad (21)$$
with \( \hat{s}_0 = \hat{s}_{365} \) and \( s_0 = s_{365} \).

3. Estimation after summer solstice until winter solstice, i.e. at \( x = 173, \ldots, 354 \):

\[
\hat{s}_x = \begin{cases} 
\hat{s}_{x-1} + 1 & \text{if } s_i > \hat{s}_{x-1} \text{ for every } i = x-5, \ldots, x-1 \\
\hat{s}_{x-1} & \text{else.}
\end{cases}
\]

(22)

Note that this algorithm estimates the lower bounds of \( s_x \). Assuming clear sky days to feature an early start of the daily feed-in, \( \hat{s}_x \) can be interpreted as the clear sky start of the daily feed-in. Thus a correction tool for the clear sky series based on \( \hat{s}_x \) seems reasonable. Estimation of the end of the daily feed-in, \( \hat{e}_x \), follows the same procedure, with inverse signs. The estimation results are again shown in Fig. 6 (solid lines). This figure also presents the estimates of the start (end) of the feed-in \( \hat{s}_{x}^{CS} \) (\( \hat{e}_{x}^{CS} \)) which originally result from the presented clear sky model (dashed lines). It can be seen that the new estimates \( \hat{s}_x \) and \( \hat{e}_x \) are much more accurate and thus more satisfactory.

![Figure 6: Actual start and end of the feed-in (gray points) and the corresponding estimates by the clear sky model (dashed lines) and the estimation algorithm (solid lines).](image)

The gap between both estimates is defined as \( \delta_{s,x} = \hat{s}_x - \hat{s}_{x}^{CS} \) for the start of the daily feed-in and \( \delta_{e,x} = \hat{e}_{x}^{CS} - \hat{e}_x \) for the end of the daily feed-in, respectively. Based on these considerations, the time series of the clear sky electricity feed-in is corrected as follows: Clear sky estimates within the correction gap are set to zero and a linear interpolation is performed on those quarter-hours that follow (precede) the corrected start (end) of the feed-in. The calculation rule for the corrected clear sky estimates for
morning quarter-hours can be written as

\[ E^{CS,C}_t = \begin{cases} 
0 & \text{if } y < \hat{s}_x \\
\frac{y - \hat{s}_x + 1}{c \delta_{x,x} + 1} & \text{if } \hat{s}_x < y < \hat{s}_x + c \delta_{x,x} \\
E^{CS}_t & \text{else},
\end{cases} \]

for each day \( x = 1 \ldots 356 \), where \( c \) is the correction procedure parameter. Evening quarter-hours are corrected accordingly. Sensitivity analysis shows that optimal results are obtained at \( c = 2 \). Choosing this value, in combination with the LM, our correction leads to an accuracy improvement of around 2% in terms of the MAE.

Fig. 7 compares uncorrected and corrected clear sky estimates along with actual values for a sample of five days that actually feature clear sky conditions. As stated at the beginning of this appendix, the original clear sky model systematically overestimates electricity feed-in around the start and the end of the daily feed-in. The presented correction algorithm significantly reduces this bias.

![Figure 7: Clear sky estimates (dashed line), corrected clear sky estimates (solid line) and actual values (gray points) of electricity feed-in for a sample of five clear sky days in 2014.](image)

Note that the bias around the start and the end of the daily feed-in is an intrinsic result of the clear sky calculation method. Due to the two-dimensional smoothing kernel, actual electricity feed-in of neighboring quarter-hours is taken into account when estimating the clear sky electricity at a given point. At a point right before (after) the start (end) of the daily feed-in, this neighborhood is fundamentally asymmetric: Subsequent (precedent) quarter-hours feature positive values whereas the lower limit of the remaining values is zero. As quantile regression does not account for these fundamental properties of the environment, quantile regression overestimates clear sky electricity feed-in at these points.
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